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NSWC TR 79-475

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ANALYTICAL MODELS FOR STRATEGIC SUBMARINE/ASW SCENARIOS

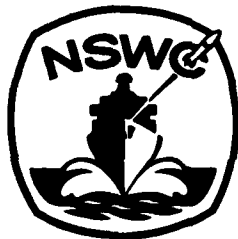
BY ELLIOT WINSTON

RESEARCH AND TECHNOLOGY DEPARTMENT

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
(14) NSWC/TR-79-475		
4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED	
(6) ANALYTICAL MODELS FOR STRATEGIC SUBMARINE/ASW SCENARIOS	(9) Status Report. Jan-Dec 1979,	
7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)	
(10) Elliot/Winston		
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS	
Naval Surface Weapons Center White Oak Silver Spring, Maryland 20910	(16) 65856N R0128SB R0128SB R44EA	
11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE	
(12) (13)	(11) 18 December 1979	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	13. NUMBER OF PAGES	
	13	
	18. SECURITY CLASS. (of this report)	
	Unclassified	
	18a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release, distribution unlimited		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
Strategic Submarine, ASW, Survivability, Target Base, Effectiveness, Game Theory		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
Several game-theoretic models of SSBN/ASW scenarios are formulated and algorithms for finding solutions are presented. No discussion of numerical studies is given.		

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SUMMARY

This report is concerned with developing mathematical models of SSBN/ASW scenarios which can be used to compare the survivability and effectiveness of present and future strategic force mixes against a variety of enemy threats. The work was supported by the Office of the Chief of Naval Operations through OP-604 under the direction of Mr. Robert Piacesi.

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1. INTRODUCTION

The policy of deterrence, adopted by the United States as a preventative to the outbreak of a major nuclear war, is currently embodied by a mix of military forces: long-range bombers, land-based intercontinental ballistic missiles, and submarine launched ballistic missiles (SLBM). The latter member of the mix is of particular interest because it is generally acknowledged to be the most survivable against an all-out attack. Although nuclear submarines are virtually undetectable by present technology, a major problem of military planning is estimating submarine survivability in the future, and the consequent effectiveness of the surviving weapons against an enemy target base. The purpose of this report is to develop some mathematical models which can be used to compare both survivability and effectiveness of present and future potential SLBM force mixes against a variety of ASW threats.

The principal mathematical tool used is game theory. In this investigation, a game consists of exactly two moves, one by each of two opposing players labeled x and y. The x-player moves first, followed by the y-player who selects his strategy with full knowledge of his opponent's play. Thus, the problem of the x-player is to find a strategy which yields minimal advantage to the y-player when y plays his best response. This is a conservative approach, and provides a "worst-case" analysis from the viewpoint of the x-player.

2. SURVIVABILITY MODEL

We begin with a derivation of the probability distribution used to describe the survivability of a single submarine which comes under attack at time $t = 0$ by y enemy units (ASW), generally taken to be aircraft. The area of the patrol zone is A. The aircraft are assumed to have no prior knowledge concerning the location of the submarine within the zone, and therefore search for it randomly. If detected, the submarine is destroyed. The motion of the submarine is approximately stationary compared to that of the aircraft over a small instant of time Δt , so that the probability of detection over this time interval is $Sy\Delta t/A$, where S is the area swept out by one aircraft during one time unit. If $p(t)$ denotes the probability that the submarine still survives at time t, then

$$p(t+\Delta t) = p(t) \left(1 - \frac{Sy\Delta t}{A} \right),$$

or

$$\frac{p(t+\Delta t) - p(t)}{\Delta t} = - \frac{Sy}{A} p(t).$$

Letting $\Delta t \rightarrow 0$ yields the differential equation

$$\dot{p}(t) = - \frac{Sy}{A} p(t).$$

The initial condition $p(0) = 1$ determines the solution

$$p(t) = e^{-Sy t/A}.$$

We are now ready to formulate the survivability model of a "surveillance-surge" scenario. The Atlantic and Pacific Oceans are each partitioned into a number of patrol zones. Submarines in one ocean cannot move over to the other, and so the number of submarines in each ocean remains constant. Surveillance does take place before the attack begins, reducing the amount of area that must be searched for a particular submarine. These "areas-of-uncertainty" have been calculated through a separate simulation procedure, and are part of input data. The problem is to determine how to assign submarines to patrol zones in such a way as to maximize the number of weapons surviving an attack by a force of aircraft which is permitted to allocate itself optimally amongst specified bases. We adopt the notation:

N_k = number of patrol zones in ocean k , $k = 1, 2$

X_k = number of submarines assigned to ocean k

Y = total number of available aircraft

A_i = area of uncertainty for a submarine in patrol zone i

T_i = time length of attack in zone i

w_i = number of weapons per submarine in zone i

x_i = number of submarines assigned to zone i

y_i = number of aircraft assigned to zone i

The expected number of surviving weapons is

$$f(x, y) = \sum_{i=1}^{N_1+N_2} w_i x_i e^{-a_i y_i},$$

where $a_i = ST_i/A_i$. If the distribution $x = (x_1, x_2, \dots, x_{N_1+N_2})$ is known to the aircraft, the appropriate response, denoted $y(x)$, is determined by

$$f(x, y(x)) = \min_y f(x, y).$$

Thus, an optimal distribution of submarines, x^* , must satisfy

$$f(x^*, y(x^*)) = \max_x f(x, y(x)) = \max_x \min_y f(x, y).$$

The complete statement of the problem is:

$$\max_x \min_y \sum_{i=1}^{N_1+N_2} w_i x_i e^{-a_i y_i}$$

under the constraints:

$$(1) \quad x_1 + \dots + x_{N_1} = X_1$$

$$(2) \quad x_{N_1+1} + \dots + x_{N_1+N_2} = X_2$$

$$(3) \quad y_1 + \dots + y_{N_1+N_2} = Y$$

$$(4) \quad x_i \geq 0, y_i \geq 0 \quad i=1,2,\dots,N_1+N_2$$

Constraints (1) and (2) conserve the number of submarines in each ocean, and constraint (3) conserves the total number of aircraft; constraint (4) eliminates meaningless solutions with negative assignments.

3. EFFECTIVENESS MODELS

Although the survivability model is of interest, it does not address the principal problem of estimating the damage capability of the surviving weapons on an enemy target base. Because its sole objective is survivability, that model might, for example, predict placing most submarines far from the target base, in effect creating an arsenal with very little targeting capability. This defect is removed by incorporating targeting into the model and altering the payoff function to measure the expected number of hard targets destroyed.

A base of targets is constructed consisting of M geographic cells, each of which contains a number, v_j , of hard targets. The probability of destroying any target within cell j from zone i is assumed to be equal to a constant value, p_{ij} . The constraint of fratricide requires that at most two weapons may be assigned to any one target. More specifically, varying assumptions on second weapon assignments give rise to two types of fratricide:

(A) the second laydown weapon assigned to a target may be launched from any zone within range;

(B) the second laydown weapon must be launched from the same zone as the first weapon.

The fraction of surviving weapons in zone i targeted to cell j in the first laydown is denoted $z_{ij}^{(1)}$; $z_{ij}^{(2)}$ is given the analogous meaning for the second laydown. The order of the player's moves is an extension of that used in the survivability model. For a known submarine distribution, the aircraft must now distribute itself in such a way as to minimize the effect of any possible targeting of the surviving weapons. The mathematical statements of the problems corresponding to the two types of fratricide are:

$$(A) \quad \max_x \min_y \max_z \sum_{j=1}^M \sum_{i=1}^{N_1+N_2} \left[w_i x_i e^{-a_i y_i} p_{ij} z_{ij}^{(1)} + \sum_{k=1}^{N_1+N_2} w_k x_k e^{-a_k y_k} p_{kj} (1-p_{ij}) z_{kj}^{(2)} \right]$$

$$(B) \quad \max_x \min_y \max_z \sum_{j=1}^M \sum_{i=1}^{N_1+N_2} w_i x_i e^{-a_i y_i} \left[p_{ij} z_{ij}^{(1)} + p_{ij} (1-p_{ij}) z_{ij}^{(2)} \right]$$

under the constraints:

$$(1) \quad x_1 + \dots + x_{N_1} = X_1$$

$$(2) \quad x_{N_1+1} + \dots + x_{N_1+N_2} = X_2$$

$$(3) \quad y_1 + \dots + y_{N_1+N_2} = Y$$

$$(4) \quad x_{ij}, y_{ij}, z_{ij}^{(1)}, z_{ij}^{(2)} \geq 0 \quad \text{all } i, j$$

$$(5) \quad \sum_{j=1}^M \left[z_{ij}^{(1)} + z_{ij}^{(2)} \right] \leq 1 \quad i=1, 2, \dots, N_1+N_2$$

$$(6) \quad \sum_{i=1}^{N_1+N_2} w_i x_i e^{-a_i y_i} z_{ij}^{(1)} \leq v_j \quad j=1, 2, \dots, M$$

$$(7A) \quad \sum_{k=1}^{N_1+N_2} w_k x_k e^{-a_k y_k} z_{kj}^{(2)} \leq \sum_{i=1}^{N_1+N_2} w_i x_i e^{-a_i y_i} z_{ij}^{(1)} \quad j=1, 2, \dots, M$$

$$(7B) \quad z_{ij}^{(2)} \leq z_{ij}^{(1)} \quad \text{all } i, j$$

Constraint (5) prevents assigning more weapons than are surviving in zone 1, constraint (6) limits the number of weapons assigned to cell j in the first laydown to be no more than the number of targets within that cell, and constraints (7A) and (7B) are the mathematical descriptions of the two types of fratricide corresponding to the payoff functions (A) and (B).

Related models occur when the players measure their respective payoffs differently. Of particular interest is the case in which the x-player maximizes the number of hard targets destroyed, and the y-player minimizes the surviving number of weapons. Mathematically, this problem is identical to the one described in the preceding paragraph except that the payoff function of the y-player is now equal to the payoff used in the survivability model.

4. SOLUTIONS

The algorithms discussed in this section provide continuous, rather than discrete, answers. Because the discrete problems are very large and complex, this has become a generally accepted practice in dealing with these types of models. The continuous answers are sometimes given probabilistic interpretations.

An algorithm, based on the notion of hedging, is used to solve the survivability problem. We visualize a series of plays of the game in which the players respond to each other alternately. Suppose the x-player initiates play with the strategy, x_0 . The y-player responds optimally with y_1 determined by

$$f(x_0, y_1) = \min_y f(x_0, y).$$

The x-player counters against y_1 with x_1 determined by

$$f(x_1, y_1) = \max_x f(x, y_1),$$

and the y-player likewise responds with y_2 . Now the x-player hedges against y_1 and y_2 , that is, he plays x_2 determined by

$$\min_{k=1,2} f(x_2, y_k) = \max_x \min_{k=1,2} f(x, y_k).$$

The play alternates in an analogous manner with x_n and y_n satisfying

$$\min_{k=1, \dots, n} f(x_n, y_k) = \max_x \min_{k=1, \dots, n} f(x, y_k)$$

and

$$f(x_n, y_{n+1}) = \min_y f(x_n, y),$$

respectively. If x^* is a limit point of $\{x_n\}$ and y^* is the optimal ASW response to x^* , then (x^*, y^*) is a saddle point solution.ⁿ For a complete theoretical discussion of hedging and its application to the survivability model, see Winston [1].

1. Winston, E., "Hedging and Maxmin," submitted for publication.

For a fixed submarine distribution, the hedging algorithm is applicable to the inner "min max" problem of the effectiveness models. In doing so, however,

$y \quad z$
modifications are made in determining the solutions of each of the hedges. The targeting portion of the algorithm is a very large linear programming problem which, if solved directly, would result in a prohibitive time and storage requirement for finding a solution of a "min max" problem. Therefore, a marginal analysis

$y \quad z$
procedure, based on an ordering, in magnitude, of the numbers $\{p_{ij}, p_{ik}(1-p_{ij})\}$, is used. Moreover, the targeting solutions determined by this method automatically satisfy all the constraints. On the other hand, constraints (6) and (7A) are non-linear in y and so are relaxed when finding the ASW hedging solutions. Although not rigorously optimal, this approximation is very fast computationally and allows the hedging algorithm to converge to good answers.

Because it is not possible to determine a closed form for $F(x) = \min \max_{y,z} f(x,y,z)$, a search method, based on a scheme proposed by Box [2], is used to maximize $F(x)$. $F(x)$ is evaluated over a random sample of size N_1+N_2 of feasible points, and the worst point is identified. The centroid of the remaining points is feasible because the constraints are linear in x . It follows that every point with non-negative components which lies on the line determined by the worst point and the centroid is also feasible. A point which is α times as far from the centroid as the reflection of the worst point through the centroid is first tested for feasibility and then compared to the worst point. This point is contracted halfway to the centroid if it is neither feasible nor an improvement over the worst point. If an admissible point is found in fewer than c contractions, it replaces the worst point, and the procedure is repeated; otherwise the best point is retained and N_1+N_2-1 new random points are generated. The process stops when the distance between a centroid and replacement point is sufficiently small.

5. SOFT TARGETING

The effectiveness models developed in Section 3 have a target base consisting of hard targets only. Naturally, the expected number of hard targets destroyed is reduced when other types of targets are also taken into account. This phenomena is observed when a soft target base, consisting of conventional military targets and economic targets, is incorporated into the model. Because the amount of hard target damage remains as the primary objective, specified amounts of soft target damage are inserted as constraints. Marginal analysis is performed on soft target damage functions of the form

$$d(x) = ve^{-x^H},$$

2. Box, M. J., "A New Method of Constrained Optimization and a Comparison to other Methods," Computer Journal, 8, 1965, pp. 42-52.

where v is the target value, and H is a fitting parameter. The first weapons used in soft targeting are those surviving weapons which have not previously been assigned to hard targets. When this stock has been depleted, weapons are removed from hard targets in the reverse order of their assignment, and reassigned to the appropriate soft targets.

6. CONCLUSION

In this report, a number of game-theoretic models of various "surveillance-surge" scenarios have been presented. The most interesting and pertinent models contain the following key features:

- (i) maximization of hard target damage,
- (ii) inclusion of a soft target base,
- (iii) consideration of fratricide constraints.

Computer codes for all models have been developed and are being exercised. The results of a series of numerical studies will appear in the classified literature.

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